

**IKKI O'LCHOVLI SIMPLEKSDA ANIQLANGAN MENDEL VA
 BERNULLI O'LCHOVLARI ORASIDAGI BOG'LANISH**

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Faraz qilaylik (E, m) - ixtiyoriy o'lchovli fazo bo'lsin. $\Omega = \bigcap_{i=1}^{\infty} E_i$ fazoni qaraymiz, bu yerda hamma i lar uchun $E_i = E$ dir. Faraz qilaylik $E = (1, 2, \dots, n)$ n ta elementdan chekli to'plam $p(i) = p_i$ - E da qaralayotgan taqsimot bo'lsin. Faraz qilaylik ξ_n funksiya, fazoning ixtiyoriy $w = (w_1, w_2, \dots) \in \Omega$ elementiga o'zining n-koordinatasi w_n ni mos qo'ysin. Bunday funksiya ξ_n n-koordinatali funksiya deyiladi.

Faraz qilaylik $F - \sigma$ -algebra hamma chekli o'lchamli t silendrlar to'plamidan hosil qilingan bo'lsin, yani quydagi to'plamlardan iborat bo'lsin

$$\{w : \xi_m(w) = i_0, \xi_{m+1}(w) = i_1, \dots, \xi_{m+k}(w) = i_k\}$$

$F - \sigma$ - algebrada ikkita P va Q o'lchovlarni qo'ydagi ko'rinishda aniqlaymiz. Q o'lchov F fazoda o'zining qiymatlari bilan bir qiymatli aniqlanadi

$$Q\{w : \xi_m(w) = i_0, \xi_{m+1}(w) = i_1, \dots, \xi_{m+k}(w) = i_k\} = Q_m(i_1, i_2, \dots, i_k) = Q_{mk}$$

Agar

$$Q_{mk} = p_{i_1} p_{i_2} \dots p_{i_k} \tag{1}$$

Deb olsak, u holda qo'ydagi munosabat o'rinli bo'ladi

$$Q_{mk} \geq 0, \quad \sum_{i=1}^n Q_m(i_1, i_2, \dots, i_k, i) = Q_{mk}, \quad \sum_{i=1}^n P_m(i) = 1$$

(1) ga mos keluvchi o'lchov Q Bernulli o'lchovi deb ataladi.

Faraz qilaylik $E = \{1, 2, 3\}$ bo'lsin. S^2 simpleksda kvadratik stoxastik operatorni qo'ydagicha aniqlaymiz:

$P_{11,1} = 1$	$P_{12,1} = 1/2$	$P_{13,1} = 0$
$P_{11,2} = 0$	$P_{12,2} = 1/2$	$P_{13,2} = 1$
$P_{11,3} = 0$	$P_{12,3} = 0$	$P_{13,3} = 0$

$$\begin{array}{lll} P_{22,1}=1/4 & P_{23,1}=0 & P_{33,1}=0 \\ P_{22,2}=1/2 & P_{23,2}=1/2 & P_{33,2}=0 \\ P_{22,3}=1/4 & P_{23,3}=1/2 & P_{33,3}=1 \end{array}$$

Bu operator uchun avloddan avlodga o'tishda Mendel qonunlari o'rinli bo'ladi. Shuning uchun bunday operatorlar Mendel operatorlari deyiladi.

Faraz qilaylik $x^{(0)} = (x_1^0, x_2^0, x_3^0)$ – E dagi boshlang'ich taqsimot va $P_x(0) = \Omega$,
 dagi qo'ydagi funksiyalar oilasiga mos keluvchi ehtimolli o'lchov bo'lsin.

$$P_{m_k} = p_m k(i_0, i_1, i_2, \dots, i_k) = x_{i_0}^{(m)} \sum_{m_1, \dots, m_k=1}^3 P_{i_0 m_1, i_1} \cdot P_{i_1 m_2, i_2} \cdot \dots$$

$$P_n(i_0, i_1, \dots, i_k) = x_{i_0}^n \sum_{m_1, \dots, m_k=1}^n P_{i_0 m_1, i_1} \cdot P_{i_1 m_2, i_2} \cdot P_{i_2 m_3, i_3} \cdot \dots \cdot P_{i_{k-1} m_k, i_k} \cdot x_{m_1}^{(n)} \cdot x_{m_2}^{(n+1)} \cdot \dots \cdot x_{m_k}^{(n+k-1)} \quad (2)$$

Chekli taqsimotlar oilasi yordamida hosil qilingan o'lchovni Mendel o'lchovi deb ataymiz.

Teoreman. Mendel o'lchovi P va Bernulli o'lchovi Q singulyardir.

Isbot. Tekshirib ko'rish qiyin emaske P_{m_k} va Q_{m_k} o'lchovlar dagi hamma shartlarni qanoatlantiradi. Shuning uchun $P \perp Q$ shart

$$\bigcap_{k \geq 0} \left(\sum_{k \geq 0} f_k^{1/2} Q_k \right) = 0 \quad (3)$$

Bo'lganda bajariladi. Bu yerda $f_n - P_{m_k}$ ni Q_{m_k} ga nisbatan Radona – Nikadima ko'paytmasidir. Shunday qilib, teoremaning isboti (3) tenglikdan kelib chiqadi. Shu tenglikni isbtlaymiz. (2) tenglikdan

$$P_{m_k} = P_m(i_0, i_1, i_2, \dots, i_k) = x_{i_0}^{(m)} \sum_{m_1=1}^3 P_{i_0 m_1, i_1} x_{m_1}^{(m)} \cdot \sum_{m_1=2}^3 P_{i_1 m_2, i_2} x_{m_1}^{(m)} \cdot x_{m_2}^{(m+1)} \cdot \dots$$

$$\sum_{m_k=1}^3 P_{i_{k-1} m_k, i_k} x_{m_{k-1}}^{(m+k-2)} x_{m_k}^{(m+k-1)} \left[x_{i_1}^{(m)} x_{i_2}^{(m+1)} \dots x_{i_{k-1}}^{(m+k-2)} \right]^{-1} =$$

$$\frac{P(\xi_k = i_0, \xi_{k+1} = i_1) \dots P(\xi_{k+k-1} = i_{k-1}, \xi_{k+k} = i_k)}{x_{i_1}^{(m)} x_{i_2}^{(m+1)} \dots x_{i_{k-1}}^{(m+k-2)}}$$

(4) ni olamiz

Bilamizki

$$P(\{w: (\xi_m(w) = 1, \xi_{m+1}(w) = 1)\}) = x_1^{(m)} \sum_{i=1}^3 P_{1i,1} x_i^{(m)} = \frac{x_1^{(m)} x_1^{(m)}}{(x_1 + 1 / 2x_2)}$$

$$P(\{w: (\xi_m(w) = 2, \xi_{m+1}(w) = 2)\}) = x_2^{(m)} \sum_{i=1}^3 P_{2i,2} x_i^{(m)} = 1 / 2x_2^{(m)}$$

$$P(\{w: (\xi_m(w) = 3, \xi_{m+1}(w) = 3)\}) = x_3^{(m)} \sum_{i=1}^3 P_{3i,3} x_i^{(m)} = \frac{x_3^{(m)} x_3^{(m)}}{(x_3 + 1 / 2x_2)}$$

$$P(\{w: (\xi_m(w) = 1, \xi_{m+1}(w) = 2)\}) = x_1^{(m)} \sum_{i=1}^3 P_{1i,2} x_i^{(m)} = \frac{x_1^{(m)} x_1^{(m)}}{2(x_1 + 1 / 2x_2)}$$

$$P(\{w: (\xi_m(w) = 1, \xi_{m+1}(w) = 3)\}) = x_1^{(m)} \sum_{i=1}^3 P_{1i,3} x_i^{(m)} = 0$$

$$P(\{w: (\xi_m(w) = 2, \xi_{m+1}(w) = 3)\}) = x_2^{(m)} \sum_{i=1}^3 P_{2i,3} x_i^{(m)} = \frac{x_2^{(m)} x_3^{(m)}}{2(x_2 + 1 / 2x_3)}$$

$$P(\{w: (\xi_m(w) = 2, \xi_{m+1}(w) = 2)\}) = 1 / 2x_2^{(m)} = 1 / 2x_2^{(1)}$$

$$P(\{w: (\xi_m(w) = 1, \xi_{m+1}(w) = 3)\}) = 0$$

dir. Qolgan hollarda

$$P(\{w: (\xi_m(w) = i_0, \xi_{m+1}(w) = i_1)\}) = \frac{x_{i_0}^{(m)} x_{i_1}^{(m)}}{2^{|i_0 - i_1|} (x_{i_0} + 1 / 2x_2)} = \frac{x_{i_0}^{(1)} x_{i_1}^{(1)}}{2^{|i_0 - i_1|} (x_{i_0} + 1 / 2x_2)}$$

(3) yordamida (4) dan

$$P_m(i_0, i_1, i_2, \dots, i_k) =$$

$$\left\{ \begin{array}{l} 0, \quad i_j=1, \quad i_{j+1}=3 \quad i_j=3, \quad i_{j+1}=1. \\ [2^{-1} x_2^{(1)}]^k \quad i_j=2 \quad j=0,1,\dots,k \\ \frac{x_{i_0}^{(1)} [x_{i_1}^{(1)} \dots x_{i_{k-1}}^{(1)}]^2 x_{i_{k-1}}^{(1)}}{4^{(|i_0-i_1|+\dots+|i_{k-1}-i_k|)} (x_{i_1} + 1/2x_2) \dots (x_{i_{k-1}} + 1/2x_2)}, \quad i_j = 1,3 \\ \frac{x_{i_0}^{(1)} [x_{i_1}^{(1)} \dots x_{i_{s+t}}^{(1)}]^2 x_2^{(1)t} [x_{i_{s+t}}^{(1)} \dots x_{i_{k-2}}^{(1)}]^2 x_{i_{k-1}}^{(1)}}{2^{2(k-1-l)} (x_{i_1} + 1/2x_2) \dots (x_{i_s} + 1/2x_2) (x_{i_{s+t}} + 1/2x_2) \dots (x_{i_{k-1}} + 1/2x_2)}, \end{array} \right. \quad (5)$$

qolgan holler uchun, bu yerda $l = |\{f : i_j = 2\}|$ to'planning quvvatini aniqlaydi. Bu yerdan osongina ko'rsatish mumkinki

$$\bigcap_{k \geq 0} \left(\sum_k (P_k / Q_k)^{1/2} Q_k \right) \leq 3 \bigcap_{k \geq 0} (x_{i_1}^{(1)} x_{i_2}^{(1)} \dots x_{i_n}^{(1)})^{1/2} = 0$$

dir. Teorema isbot bo'ldi.

Natija. $x^{(0)} \neq \bar{x}^{(0)} \in S^2$ bo'lganda $P_{x^{(0)}}$ o'lchov va $P_{\bar{x}^{(0)}}$ o'lchov singulyardir.

Adabiyotlar

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