ANIQ INTEGRALLARNI TAQRIBIY HISOBLASHNING BA'ZI USULLARI

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Biz aniq integralni hisoblashda Nyuton Leybnits formulasi yordamida topiladi. Lekin ba'zi bir funksiyalarni berildan oraliqda boshlang'ich funksiyasini topish bir muncha qiyinchilik tug'diradi. Masalan: $\int \frac{\sin x}{x}dx, \int \sin x^2\,dx, \int \frac{dx}{\ln x}$.... kabi funksiyalarning boshlang'ich funksiyasini hamma vaqt ham elementar funksiyalar orqali ifodalab bo'lmaydi. Shuning uchun biz bu integralllarni hisoblash uchun bir qancha formulalarni ko'rib o'tamiz.

Aniq integralni taqribiy hisoblashda biz to'g'ri to'rtburchak,parabola (Simpson),trapetsiyalar formulasi kabi usullarni qanday qo'llanilishini va ba'zi bir aniq integrallarni hisoblamasdan natija olish usullarini ham ko'rib o'tamiz.

Aniq integralni taqribiy hisoblash usullari

1) To'g'ri to'rtburchak usuli:

Bizga $f(x)$ funksiya $[a,b]$ berilgn va uzluksiz bo'lsin. Endi shu $f(x)$ funksiyaning aniq integrali $\int_a^b f(x)\,dx$ ni taqribiy hisoblash formulasini keltiramiz.

Hisoblashlarda aniq integralni yuzini ifodalovchi yig'indi limiti deb, ya'ni $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(\xi)(x_i - x_{i-1})$ ko'rinishda mulohaza yuritiladi. Endi esa $[a,b]$ kesmani $a = x_0, x_1, x_2, ..., x_n = b$ nuqtalar bilan teng n ta bo'lakka bo'lamiz. Har birining uzunligini $h = \frac{b-a}{n}$, $x_i = a + ib\ (i = \overline{0,n})$ deb olamiz. $x = x_i$ bo'lganda $f(x)$ funksiya qiymatlarini $y_i = f(x_i) = f(a+ih)$ deb belgilaymiz.

$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(\xi)(x_i - x_{i-1})$ o'ng tomonidagi yig'indini $\xi_i = x_{i-1}$ yoki x_i deb, quydagi ikkita formulani hosil qilamiz.

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[y_0 + y_1 + ... + y_{n-1}\right] \qquad (1)$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[y_1+y_2+...+y_n\right] \qquad (2)$$

Yuqorida keltirilgan (1) va (2) formulalar aniq integralni hisoblashning to'g'ri to'rtburchak formulasi deyiladi.1-chizmada quydagilar tasvirlangan: agar $f(x)$ musbat va o'suvchi funksiya bo'lsa, u holda (1) formula "ichki" to'g'ri to'rtburchaklardan tuzilgan zinapoyasimon shaklning yuzini tasvirlaydi. (4) formula esa "tashqi" to'g'ri to'rtburchaklardan tuzilgan zinapoyasimon shaklning yuzini tasvirlaydi. Integralni to'g'ri to'rtburchak formulasi bilan hisoblashda qilingan xato n son qancha katta(ya'ni bo'linish qadami h qancha kichik) bo'lib borishi (1) va (2) formulalar aniqroq bo'lib boradi, ya'ni $n\to\infty$ da $h\to\infty$ da ular aniq integralning haqiqiy qiymatini beradi.

2) **Trapetsiya formulasi.**

Agar ordinatalar chizig'ining egri chiziq bilan kesishgan nuqtalarini zinapoyali siniq chiziqlar bilan emas, balki ichki chizilgan siniq chiziqlarbolan tutashtirsak, (1) va (2) formulalarga nisbatan xatosi kamroq bo'lgan taqribiy formulani keltirib chiqaramiz:

Bu holda egri chiziqli $aABb$ trapetsiyaning yuzi yuqoridan $AA_1, A_1A_2,...,$ $A_{n-1}B$ vatarlar bilan chegaralangan to'g'ri chiziqli trapetsiyalar yuzalarining yig'indisiga teng bo'ladi. Natijada trapetsiyalar formulasini hosil qilamiz:

$$\int_a^b f(x)\,dx \approx h\left[\frac{y_0+y_1}{2}+\frac{y_1+y_2}{2}+...+\frac{y_{n-1}+y_n}{2}\right]=h\left[\frac{y_0+y_n}{2}+y_1+y_2+...+y_{n-1}\right]$$

(3)

bunda

$$h=\frac{b-a}{n},\ x_i=x_{i-1}+h,\ (i=\overline{1,n}),\ x_0=a,\ \ x_n=b$$

(3) ga trapetsiyalar formulasi deyiladi.

3) **Parabolalar (Simpson) formulasi.**

$[a,b]$ kesmani juft sonda $n=2m$ bo'lakka ajratamiz. $[x_0,\ x_1]$ va $[x_1,\ x_2]$ kesmalarga mos va berilgan $y=f(x)$ egri chiziq bilanchegaralangan egri chiziqli trapetsiyaning yuzini $M(x_0,y_0)$, $M_1(x_1,y_1)$, $M_2(x_2,y_2)$, uchta nuqtadan o'tuvchi va o'qi Oy o'qi parallel bo'lgan ikkinchi darajali parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzi bilan almashtiramiz.

Bunday egri chiziqli trapetsiyani parabolic trapetsiya deb ataymiz.

O'qi Oy o'qiga parallel bo'lgan parabolaning tenglamasi $y=Ax^2+Bx+C$ (4) ko'rinishda bo'ladi. A, B va C koeffisentlar parabolaning berilgan uch nuqta orqali o'tishshartidan bir qiymatli ravishda aniqlanadi. Shunday yasalgan parabolik trapetsiyalar yuzalarining yig'indisi integralning taqribiy qiymatini beradi.

Lemma. Agar egri chiziqli trapetsiya (4) parabola, Ox o'q va oralig'i $2h$ ga teng bo'lgan ikkita ordinata bilan chegaralangan bo'lsa, u holda uning yuzi

$$S=\frac{h}{3}(y_0+y_1+y_2) \qquad (5)$$

ga teng, bunday y_0 va y_2 chetdagi ordinatalar y_1 esa egri chiziqning kesma o'rtasidagi ordinatasi.

(5) formuladan foydalanib, quydagi taqribiy qiymatlarni yozamiz.

$$\int_{a=x_0}^{x_2} f(x)\,dx \approx \frac{h}{3}(y_0+4y_1+y_2)$$

$$\int_{x_2}^{x_4} f(x)\,dx \approx \frac{h}{3}(y_2+4y_3+y_4)$$

$$\int_{x_{2(m-1)}}^{x_{2m=b}} f(x)\,dx \approx \frac{h}{3}(y_{2(m-1)}+4y_{2m-1}+y_{2m})$$

Yuqoridagi taqribiy qiymatlarning chap va o'ng tomonlarini qo'shib, chapda izlanayotgan integralni, o'ngda esa unig taqribiy qiymatini hosil qilamiz:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}(y_0+4y_1+2y_2+4y_3+...+2y_{2(m-1)}+4y_{2m-1}+y_{2m}) \qquad (6)$$

yoki

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6m}\left[y_0+y_{2m}+2(y_2+y_4+...+y_{2m-2})+4(y_1+y_3+...+y_{2m-1})\right]$$

(7)

(7) formulaga Simpson formulasi deyiladi. Bu yerda bo'linish nuqtalarining soni $2m$ ixtiyoriy, lekin bu son qancha katta bo'lsa, (7) tenglik o'ng tomonidagi yig'indi integralning qiymatini shuncha aniq ifodalaydi.

Endi esa taqribiy hisoblashga oid ba'zi bir aniq integrallarni ko'rib o'tamiz:

Misol: $\int_0^1 \frac{dx}{1+x^2}$ integralning qiymati $n=10$ bo'lganda taqribiy hisoblansin.

Yechish: $h=\frac{b-a}{n}$ formulaga ko'ra $a=0, b=1, n=10$ ekanligidan

$$h=\frac{1-0}{10}=0{,}1$$

Demak, $h=0{,}1$ bo'ladi.

Endi o'zimizga qulay bo'lishi uchun quydagi jadvalni tuzib olamiz.

x	$1+x^2$	y_0, y_{10}	$y_1, y_3, ..., y_9$	$y_2, y_4, ..., y_8$
0	1	1		
0,1	1,01		0,9901	
0,2	1,04			0,9615
0,3	1,09		0,9174	
0,4	1,16			0,8621

0,5	1,25		0,8	
0,6	1,36			0,7353
0,7	1,49		0,6711	
0,8	1,64			0,6098
0,9	1,81		0,5525	
1	2	0,5		
		$\sum_0=1,5$	$\sum_1=3,9311$	$\sum_2 3,1687$

1) Chap to'g'ri to'rtburchak formulasi:

$$\int_0^1 \frac{dx}{1+x^2} \approx h\left(y_0+\sum_1+\sum_2\right)=0,1(1+3,9311+3,1687)\approx 0,81$$

2) O'ng to'g'ri to'rtburchak formulasi:

$$\int_0^1 \frac{dx}{1+x^2} \approx h\left(\sum_1+\sum_2+y_{10}\right)=0,1(3,9311+3,1687+0,5)\approx 0,76$$

3) Trapetsiyalar formulasi:

$$\int_0^1 \frac{dx}{1+x^2} \approx h\left(\frac{\sum_0}{2}+\sum_1+\sum_2\right)=0,1\left(\frac{1,5}{2}+3,9311+3,1687\right)\approx 0,785$$

4) Simpson formulasi:

$$\int_0^1 \frac{dx}{1+x^2} \approx \frac{h}{3}\left(\sum_0+4\sum_1+2\sum_2\right)=\frac{0,1}{3}(1,5+4\cdot 3,9311+2\cdot 3,1687)\approx$$

$$\approx \frac{0,1}{3}(1,5+15,7244+6,3374)\approx \frac{0,1}{3}\cdot 23,5618\approx 0,7854$$

Endi integralni o'zimiz bilgan usulda hisoblab ko'ramiz va olingan natijalarni taqqoslaymiz:

$$\int_0^1 \frac{dx}{1+x^2} = arctgx\Big|_0^1 = arctg1-arctg0=\frac{\pi}{4}-0=\frac{\pi}{4}\approx 0,7854$$

Demak, yuqorida keltirilgan usullardan Simpson formulasi yordamida taqribiy hisoblash aniq integralning aniqroq qiymatiga erishadi.

Endi esa yuqorida keltirilgan usullarning yechilishini maple dasturi yordamida hisob ko'ramiz:

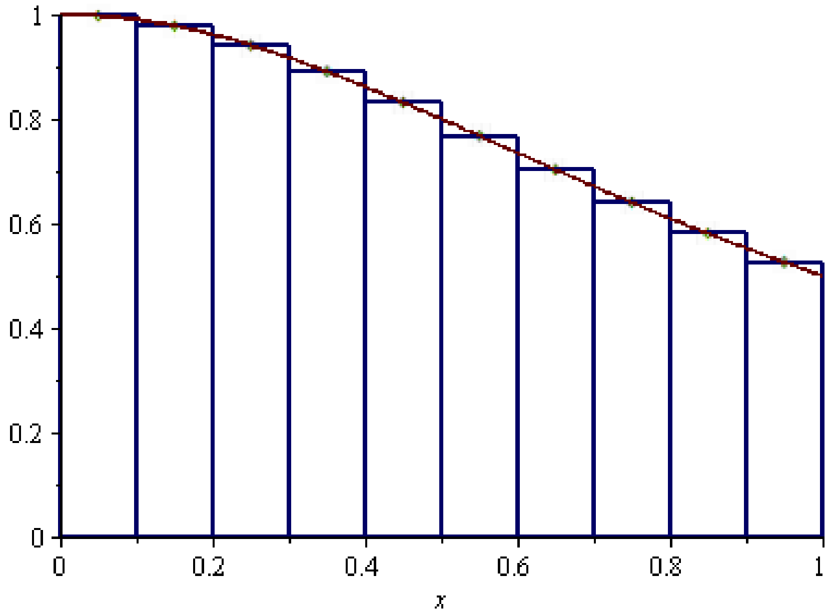
1) Chap to'g'ri to'rtburchak formulasi:

```
> restart;
with(Student[Calculus1]) :
RiemannSum(1/(1 + x^2), x = 0 ..1, method = left); evalf(%);
```

$$\frac{1579799420518583}{1950414208136225}$$

0.8099814972

$$RiemannSum\left(\frac{1}{1+x^2}, x = 0..1, output = plot\right);$$



A midpoint Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = \frac{1}{x^2+1}$ and the partition is uniform. The approximate value of the integral is 0.7856064964 . Number of subintervals used:10.

2) O'ng to'g'ri to'rtburchak formulasi:

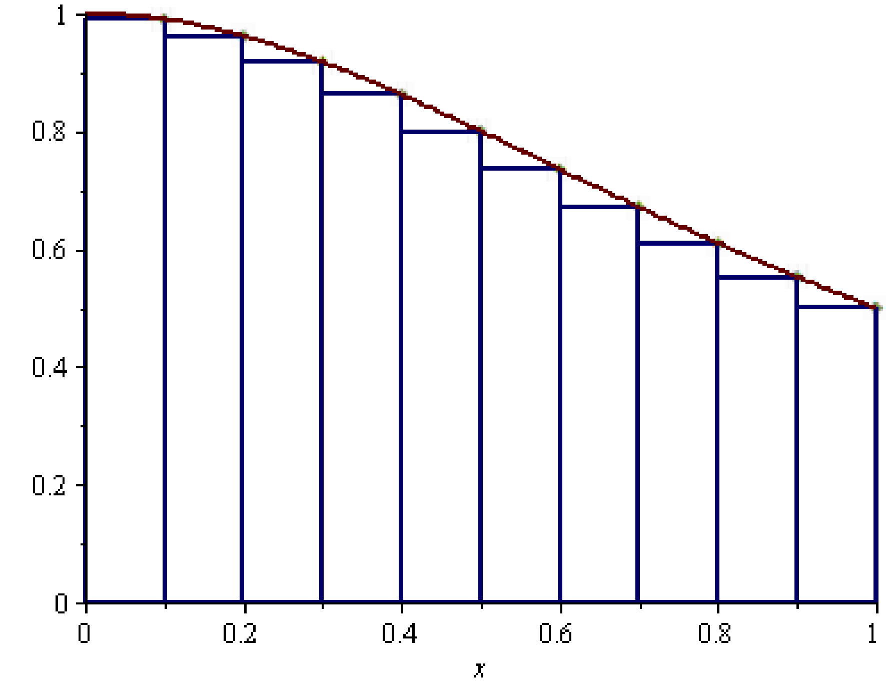
$> with(Student[Calculus1]):$

$$RiemannSum\left(\frac{1}{1+x^2}, x = 0..1, method = right\right); evalf(\%);$$

$$\frac{5929114840447087}{7801656832544900}$$

$$0.7599814972$$

$$RiemannSum\left(\frac{1}{1+x^2}, x = 0..1, method = right, thickness = 2, output = plot\right);$$



A right Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = \frac{1}{x^2+1}$ and the partition is uniform. The approximate value of the integral is 0.7599814974 . Number of subintervals used:10.

3) Trapetsiyalar formulasi:

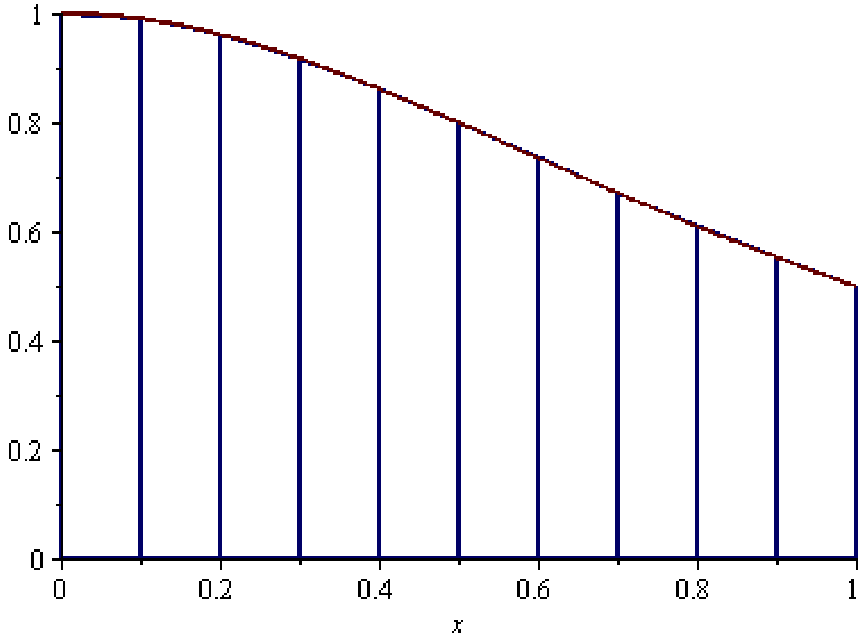
> $with(Student[Calculus1])$:

$$ApproximateInt\left(\frac{1}{1+x^2}, x = 0..1, method = trapezoid\right); evalf(\%);$$

$$\frac{12248312522521419}{15603313665089800}$$

0.7849814972

$$ApproximateInt\left(\frac{1}{1+x^2}, x = 0..1, method = trapezoid, output = plot\right);$$



An approximation of $\int_0^1 f(x)\,dx$ using trapezoid rule, where $f(x) = \frac{1}{x^2+1}$ and the partition is uniform. The approximate value of the integral is 0.7849814972 . Number of subintervals used:10 .

4) Simpson formulasi:

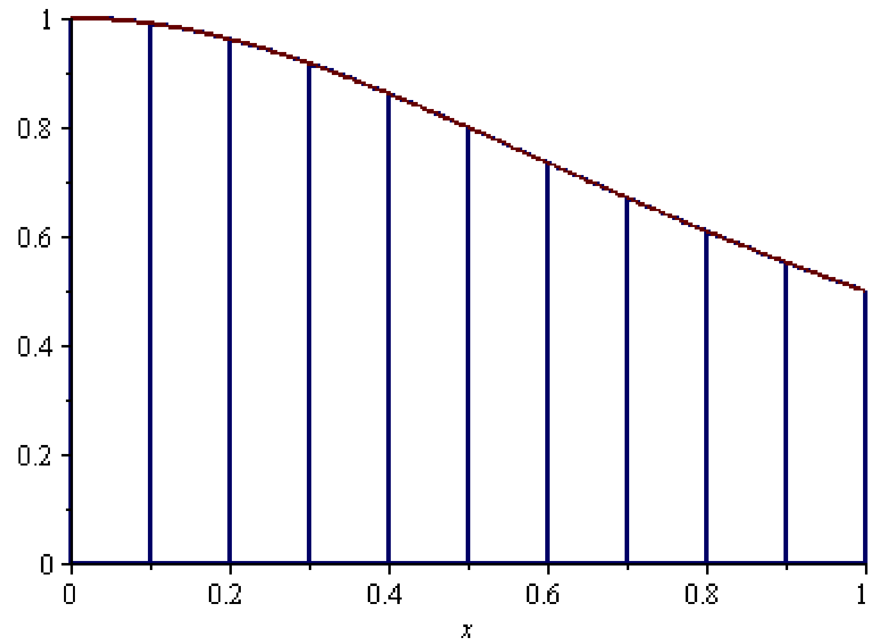
> $with(Student[Calculus1])$:

$ApproximateInt\left(\frac{1}{1+x^2}, x = 0..1, method = simpson\right); evalf(\%);$

$$\frac{5988585315838311774901484536676836463}{7624903642650463520301694141655283000}$$

0.7853981632

$ApproximateInt\left(\frac{1}{1+x^2}, x = 0..1, method = simpson, output = plot\right);$



An approximation of $\int_0^1 f(x)\,dx$ using Simpson's rule, where $f(x) = \frac{1}{x^2+1}$ and the partition is uniform. The approximate value of the integral is 0.7853981632 . Number of subintervals used:10.

Foydalanilgan adabiyotlar

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