

## BIR JINSLI BO'LMAGAN STERJENDA ISSIQLIK ALMASHISH TENGLAMASI UCHUN QO'YILGAN BOSHQARUV MASALASI

*Quldoshova Maftuna J*

*TKTI Shahrisabz filiali o'qituvchisi*

**Anotatsiya.** Soha chegarasining bir qismida harorati boshqariladigan isitgich joylashgan. Sohaning qism sohasida o'rta haroratni ta'minlaydigan isitgichning ishlash jarayonini topish talab qilinadi. Bu masalaning matematik modeli tuzilib, uning yechimining mavjudligi va yagonaligi isbotlangan.

**Kalit so'zlar:** Parabolik tipdagi tenglama, boshlang'ich shart, chegaraviy shart, o'rtacha temperatura, boshqaruv masalasi.

**Annotation.** We consider the following problem. Through parts of the border of are a there is heater having regulate temperature. It is required to find such operating heater, that average temperature in some subarea D of the area accepted the given value. The theorems of existence and uniqueness of the solution are proved.

**Keywords:** parabolic type equation, initial condition, boundary condition, temperature, existence

**Аннотация** :Рассмотрим следующую задачу. Через части бордюра проходит нагреватель, имеющий регулировку температуры. Требуется найти такой работающий обогреватель, чтобы средняя температура в некоторой подзоне района принимала заданное значение. Доказаны теоремы существования и единственности решения.

**Ключевые слова:** Уравнение параболического типа, начальное условие, граничное условие, средняя температура, задача управления.

Quyidagi sohada  $D = \{(x, t) : 0 < t < T, 0 < x < l\}$  issiqlik tarqalish tenglamasi va chegaraviy va boshlang'ich shartlarni qaraylik.

$$u_t(x, t) = \frac{\partial}{\partial x} \left( k(x) \frac{\partial u(x, t)}{\partial x} \right) + f(x, t), \quad 0 < x < l; t > 0;$$

(1)

$$u(0, t) = \mu(t), \quad u(l, t) = 0; \quad (2)$$

$$u(x, 0) = \varphi(x). \quad (3)$$

bu yerda  $k(x)$  sterjen materiali tarkibini xarakterlovchi funksiya,  $f(x, t)$  esa sterjen ichida issiqlik chiqaruvchi yoki yutuvchi manba funksiyasi,  $\mu(t)$  sterjen chetida issiqlikni boshqarish funksiyasi, funksiya  $\varphi(x)$  -sterjenning boshlang'ich holati.

(2) va (3) shartlardan

$$u(0, 0) = \varphi(0) = \mu(0), \quad u(l, 0) = \varphi(l) = 0. \quad (4)$$

shartlar kelib chiqadi.

Bundan keyin boshqaruv funksiyasi  $\mu(t)$  funksiyani chegaralangan deb faraz qilamiz:

$$|\mu(t)| \leq M.$$

(5)

Sterjenning o'rtacha temperaturasi

$$\frac{1}{l} \int_0^l u(x,t) dx = b(t)$$

(6)

kabi belgilaymiz.

**Masalaning qo'yilishi.**  $b(t)$  funksiya berilgan bo'lsin. U holda shunday  $\mu(t)$  boshqaruv funksiyasini topish kerakki (1)-(3) masala yagona yechimga ega bo'lsin va bu yechim (6) shartni qanoatlantirsin.

Ushbu masalani yechishda berilgan funksiyalar uchun quyidagi talablarni qo'yamiz.  $k(x) \in C^1[0;l], k(x) > k_0 > 0$ ,

**Teorema.** Berilgan funksiyalarimiz quyidagi sinfga tegishli bo'lsin .  $k(x) \in C^1[0;l], k(x) > k_0 > 0, \varphi \in C[0,l] f(x,t) \in C[0;l] \times C(R_+)$  va  $\mu \in C^1[0, +\infty]$ . U holda (1)-(4) masalaning yechimi mavjud va yagona.

**Isbot.**

$$\int_0^l \varphi(x) dx = 0.$$

(1)-(3) masalani yechish uchun quyidagi xos qiymat masalasini qaraymiz:

$$-\frac{d}{dx} \left( k(x) \frac{d\varphi_n(x)}{dx} \right) = \lambda_n^2 \varphi_n(x), \quad \varphi_n(0) = \varphi_n(l) = 0. \quad (7)$$

Differensial operatorlarning spektral nazaryasi kursidan ma'lumki (Levitan.B.M) (7) xos qiymat masalasi yechimga ega va bu masalaning yechimlari bo'lgan  $\{\varphi_n(x)\}$  xos funksiyalari sistemasi  $L_2[0;l]$  fazoda ortonormal bazis tashkil qiladi ya'ni:

$$\int_0^l \varphi_n(y) \varphi_m(y) dy = \delta_{nm} = \begin{cases} 1, & n = m, \\ 0, & n \neq m. \end{cases}$$

Agar quyidagi ko'rinishdagi

$$G(x, y; t) = \sum_{n=1}^{\infty} e^{-\lambda_n^2 t} \varphi_n(x) \varphi_n(y) \quad (8)$$

Grin funksiyasini kiritsak (1)-(3) masalaning yechimi

$$u(x, t) = \omega(x, t) + \int_0^t \int_0^l G(x, \xi; t - \tau) \left[ \frac{\partial}{\partial \xi} \left\{ k(\xi) \frac{\partial \omega(\xi, \tau)}{\partial \xi} \right\} - \frac{\partial \omega(\xi, \tau)}{\partial \tau} \right] d\xi d\tau +$$

(9)

$$- \int_0^l G(x, \xi; t) \omega(\xi, 0) d\xi + \int_0^t \int_0^l G(x, \xi; t - \tau) f(\xi; \tau) d\xi d\tau + \int_0^l G(x, \xi; t) \varphi(\xi) d\xi$$

ko'rinishda bo'ladi.

Bevosita Grin funksiyasining (8) ko'rinishidan quyidagi tasdiqning o'rinligi kelib chiqadi.

**Лемма 1.**

- 1)  $G(x, y; t) = G(y, x; t);$
- 2)  $\frac{\partial G(x, y; t)}{\partial t} = \frac{\partial}{\partial x} \left( k(x) \frac{\partial G(x, y; t)}{\partial x} \right);$
- 3)  $G(0, y; t) = G(l, y; t) = 0;$
- 4)  $G(x, 0; t) = G(x, l; t) = 0;$
- 5)  $G(x, y; 0) = \delta(x - y).$

Lemma.1 dagi oxirgi 5) xossani umumlashgan ma'noda tushunamiz, bu yerda  $\delta(x)$  Dirakning "delta" funksiyasi.

$$\text{Demak, } u(x, t) = k(0) \int_0^t \mu(\tau) \frac{\partial}{\partial \xi} G(x, 0; t - \tau) d\tau + \int_0^t \int_0^l G(x, \xi; t - \tau) f(\xi; \tau) d\xi d\tau + \int_0^l G(x, \xi; t) \varphi(\xi) d\xi.$$

(10)

Oldimizga qo'yilgan masalani yechish uchun (13) tenglikni integrallaymiz.

$$\int_0^t \mu(\tau) \left\{ \frac{1}{l} k(0) \int_0^l \frac{\partial}{\partial \xi} G(x, 0; t - \tau) dx \right\} d\tau = b(t) - \int_0^t \left\{ \frac{1}{l} \int_0^l \int_0^l G(x, \xi; t - \tau) f(\xi; \tau) d\xi dx - \frac{1}{l} \int_0^l \int_0^l G(x, \xi; t) \varphi(\xi) d\xi dx \right\} d\tau.$$

Quyidagi mulohazalarimizni qulaylashtirish maqsadida ba'zi belgilashlarni kiritamiz.

$$K(t - \tau) = \frac{1}{l} k(0) \int_0^l \frac{\partial}{\partial \xi} G(x, 0; t - \tau) dx = \frac{1}{l} \int_0^l \left\{ k(\xi) \frac{\partial}{\partial \xi} G(x, \xi; t - \tau) \Big|_{\xi=0} \right\} dx$$

$$F(t) = - \int_0^t \left\{ \frac{1}{l} \int_0^l \int_0^l G(x, \xi; t - \tau) f(\xi; \tau) d\xi dx \right\} d\tau,$$

$$\Phi(t) = - \int_0^t \left\{ \frac{1}{l} \int_0^l \int_0^l G(x, \xi; t) \varphi(\xi) d\xi dx \right\} d\tau.$$

U holda biz quyidagi Volterra tenglamasiga kelamiz.

$$\int_0^t \mu(\tau) K(t - \tau) d\tau = b(t) + F(t) + \Phi(t) \tag{11}$$

(11) integral tenglama yechimga ega bo'lishligi uchun  $b(0) + F(0) + \Phi(0) = 0$  bo'lishi zarur. Buning uchun  $b(0) = 0$  shartning bajarilishi yetarli. Bu shart bizga sterjenning boshlang'ich sharti uchun quyidagi zaruriy shartni keltirib chiqaradi:

$$b(0) = \frac{1}{l} \int_0^l u(x, 0) dx = \frac{1}{l} \int_0^l \varphi(x) dx = 0.$$

(11) tenglamani yechish uchun Laplas almashtirishini qo'llaymiz, ya'ni quyidagi almashtirishni qaraymiz:

$$\int_0^{+\infty} e^{-pt} \left( \int_0^t \mu(\tau) K(t - \tau) d\tau \right) dt = \int_0^{+\infty} e^{-pt} \{ b(t) + F(t) + \Phi(t) \} dt,$$

bu yerda  $p = a + is$  -kompleks son. U holda

$$\begin{aligned} \int_0^{+\infty} e^{-pt} \left( \int_0^t \mu(\tau) K(t - \tau) d\tau \right) dt &= \int_0^{+\infty} \mu(\tau) d\tau \int_{\tau}^{+\infty} e^{-pt} K(t - \tau) dt = \\ &= \int_0^{+\infty} \mu(\tau) e^{-p\tau} d\tau \int_{\tau}^{+\infty} e^{-p(t-\tau)} K(t - \tau) dt = \int_0^{+\infty} \mu(\tau) e^{-p\tau} d\tau \int_0^{+\infty} e^{-p\lambda} K(\lambda) d\lambda = \\ &= \widehat{\mu}(p) \widehat{K}(p). \end{aligned}$$

$$\int_0^{+\infty} e^{-pt} \{ b(t) + F(t) + \Phi(t) \} dt = \widehat{b}(p) + \widehat{F}(p) + \widehat{\Phi}(p).$$

$$\widehat{\mu}(p) = \frac{\widehat{b}(p) + \widehat{F}(p) + \widehat{\Phi}(p)}{\widehat{K}(p)}. \tag{12}$$

Agar (12) tenglikdan Mellin almashtirishini bajarib  $\mu(t)$  funksiyani topamiz.

$$\mu(t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \frac{\widehat{b}(p) + \widehat{F}(p) + \widehat{\Phi}(p)}{\widehat{K}(p)} e^{pt} dp =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\widehat{b}(a+is) + \widehat{F}(a+is) + \widehat{\Phi}(a+is)}{\widehat{K}(a+is)} e^{(a+is)t} ds.$$

$$\widehat{K}(p) = \int_0^{+\infty} e^{-pt} K(t) dt = \int_0^{+\infty} e^{-pt} \left( \frac{1}{l} k(0) \int_0^l \frac{\partial}{\partial \xi} G(x, 0; t) dx \right) dt =$$

$$= \frac{1}{l} k(0) \sum_{n=1}^{\infty} \varphi'_n(0) \int_0^l \varphi_n(x) dx \int_0^{+\infty} e^{-pt - \lambda_n^2 t} dt,$$

$$\widehat{K}(p) = \frac{1}{l} k(0) \sum_{n=1}^{\infty} \frac{1}{p + \lambda_n^2} \varphi'_n(0) \int_0^l \varphi_n(x) dx.$$

Xuddi shunga o'xshash  $\widehat{F}(p)$ ,  $\widehat{\Phi}(p)$  funksiyalarni topamiz.

$$\widehat{F}(p) = - \int_0^{+\infty} e^{-pt} \int_0^t \left\{ \frac{1}{l} \int_0^l \int_0^l G(x, \xi; t - \tau) f(\xi; \tau) d\xi dx \right\} d\tau dt =$$

$$= - \frac{1}{l} \sum_{n=1}^{\infty} \int_0^l \int_0^l \varphi_n(\xi) \varphi_n(x) \frac{\widehat{f}(\xi; p)}{p + \lambda_n^2} d\xi dx.$$

$$\Phi(p) = - \frac{1}{l} \int_0^l \int_0^l g(x, \xi; p) \widehat{f}(\xi; p) d\xi dx.$$

$$g(x, \xi; p) = \sum_{n=1}^{\infty} \frac{\varphi_n(x) \varphi_n(\xi)}{p + \lambda_n^2}.$$

(11) tenglama birinchi tur Volterra tenglamasi shu sababli (11) tenglikning ikkala tomonidan  $t$  bo'yicha hosila olamiz. Natijada

$$K(0)\mu(t) + \int_0^t \mu(\tau) K'(t - \tau) d\tau = b'(t) + F'(t) + \Phi'(t)$$

(12)

(12) tenglamada  $K(0) \neq 0$  bo'lsa (15) tenglama ikkinchi tur Volterra tenglamasi bo'ladi. Ma'lumki ikkinchi tur Volterra tenglamasi har doim yechimga ega. Bu holda masala to'liq yechildi.

### Adabiyotlar

1. SH. Alimov, On A Control Problem Associated With The Heat Transfer Process. Eurasian mathematical journal, Number 2 (2010), 17-30.
2. S. Albeverio, SH. Alimov, On a time-optimal control problem associated with the heat exchange process. Applied Mathematics and Optimization, 57, no. 1 (2008), 58-68.
3. Тихонов А.Н., Самарский А.А. Уравнения математической физики. М.: наука. 1966А.
4. К.Фаязова. Граничное управление процессом теплообмена. Узбекский математический журнал. 2013, №2 стр. 184-186.

5. Ю.Э.Файзиев, Н.Халилова. О задаче управления процессом теплопроводности. Вестник НУз. 2016, С.49-54.