CONSTRUCTION OF A SELF-SIMILAR SOLUTION OF A NONLINEAR HEAT EQUATION IN A VARIABLE MEDIUM WITH A DENSITY OF TIME AND OBTAINING NUMERICAL RESULTS

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Annotation. in the thesis, an analysis of new properties of a mathematical model is carried out based mainly on theorems of studying non-linear processes, comparison. The results are given in animation and graphic representation. Appropriate methods and algorithms, calculation schemes are proposed.

Keywords: self-similar, diffusion, density, ambient speed, thermal conductivity, algorithm, calculation schemes.

Putting the matter. $Q=\{(t,x): 0<t\le T, a<x<b\}$ in the field let's look at the boundary problem for the nonlinear parabolic equation in the environment with the following variable properties:

$$\rho(t)\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{\sigma}\frac{\partial u}{\partial x}\right)-\vartheta(t)\frac{\partial u}{\partial x} \qquad (1)$$

$$u(x,0)=u_0(x)\ge 0, a\le x\le b \quad (2)$$

$$\begin{cases} u(t,a)=\phi_1(t) \\ u(t,b)=\phi_2(t) \end{cases}, 0\le t\le T \qquad (3)$$

here $\sigma\in R_+$-given a real number $\vartheta(t)$- the speed of the environment, u^{σ}-thermal conductivity koeffisienti $\sup mes\, u(x,t)<\infty$,$u=u(x,t)\ge 0$ wanted, or $u_0(x)$-the initial temperature, $\phi_1(t),\phi_2(t)$-function, which is negative, T, a, b-given the number (1) many equations-dimensional linear distribution and in the field of the environment are changing the speed and density of heat, are linear heat conductivity, diffusion balances, liquid and gas filtration minds investigate processes such as the movement of the water under the earth, and (2) Cauchy condition (3) is called the boundary conditions. There are several ways to solve this equation, from which the solution of the equation through a self-similar solution is considered. Nowadays, in the research of non-linear heat dissipation processes, the concept of a self-similar solution has gained very wide popularity.

(1) the equation represents a series of physical processes: in a nonlinear medium, the reaction represents the diffusion process, the heat dissipation process in a non-homogeneous nonlinear medium, the law of polytrophic and the existence of other nonlinear migrations.

The solution to the following equation (1) we look for $u(t,x)$in the form $u(t,x)=V(\tau(t),x)$ (*). We put (*) to (1) and the following:

$\rho(t)\frac{\partial V}{\partial \tau}\tau' = \frac{\partial}{\partial x}\left(V^{\sigma}\frac{\partial V}{\partial x}\right) - Z(\tau)\frac{\partial V}{\partial x}$ (4)

an equation is formed. Where $Z(\tau)$: $t \to \tau, \vartheta(t) \to Z(\tau)$ is formed by transfer.

$$\tau'(t)\rho(t) = 1 \ , \tau(0) = 0 \quad (5)$$

an equation is formed and from this equality for $\tau(t)$ we form the following equality.

$$\tau'(t) = \frac{1}{\rho(t)}$$

$\tau(t) = C + \int \frac{1}{\rho(t)}dt$ (6)

We will have equality. We put (6) in (4) and equation (4) goes to the following view:

$\frac{\partial V}{\partial \tau} = \frac{\partial}{\partial x}\left(V^{\sigma}\frac{\partial V}{\partial x}\right) - Z(\tau)\frac{\partial V}{\partial x}$ (7)

(7) to find the solution of the equation $V(\tau(t), x)$:

$$V(\tau, x) = W\big(\tau, \gamma(\tau, x)\big) \quad (8)$$

visible replacement will be performed.

We put (8) in equation (7) and perform the necessary calculations.

$$W_\tau + \gamma_\tau W_\gamma = \frac{\partial}{\partial x}\left(\gamma_x W^{\sigma} W_\gamma\right) - Z(\tau)\gamma_x W_\gamma \qquad (9)$$

$\gamma_\tau W_\gamma = -Z(\tau)\,\gamma_x W_\gamma$ (10)

$$\gamma_\tau = -Z(\tau)\gamma_x$$

$$-\frac{\gamma_\tau}{Z(\tau)} = \gamma_x$$

$\gamma(\tau, x) = x - \int_0^\tau Z(\theta)d\theta$ (11)

We put (11) in (9) and form the following equality:

$$W_\tau = \frac{\partial}{\partial x}\left(\gamma_x W^{\sigma} W_\gamma\right) \qquad (12)$$

(12) we also carry out the necessary calculations for:

$W_\tau = \frac{\partial}{\partial x}(\gamma_x)\left(W^{\sigma}W_\gamma\right) + \gamma_x\frac{\partial}{\partial x}\left(W^{\sigma}W_\gamma\right)$ (13)

$\gamma_x = 1 => \frac{\partial}{\partial x}(\gamma_x) = 0$ (14)

We put (14) in (13) and form the following equation:

$W_\tau = \gamma_x\frac{\partial}{\partial x}\left(W^{\sigma}W_\gamma\right)$ (15)

$$\frac{\partial}{\partial x}\left(W^{\sigma}W_\gamma\right) = \frac{\partial}{\partial \gamma}\left(W^{\sigma}W_\gamma\right)\gamma_x$$

$$W_\tau = (\gamma_x)^2\frac{\partial}{\partial x}\left(W^{\sigma}W_\gamma\right)$$

$\frac{\partial W}{\partial \tau} = \frac{\partial}{\partial \gamma}\left(W^{\sigma}W_\gamma\right)$ (16)

We build a self-similar solution to find the following (16) equation solution $W(\tau, \gamma(\tau, x))$. In doing so, we look for the solution as follows:

$W\big(\tau, \gamma(\tau, x)\big) = (T - \tau)^{\alpha} f(\xi)$ (17)

$\xi = \gamma(T-\tau)^{-\beta}$ (18)

We put (17) and (18) (16) and transfer (16) to the following view:

$\frac{\partial W}{\partial \tau} = \frac{\partial}{\partial \tau}\left(\left(T-\tau\right)^{\alpha} f(\xi)\right) = -\alpha\, f(\xi)(T-\tau)^{\alpha-1} + (T-\tau)^{\alpha}\frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial \tau}$ (19)

$$\frac{\partial \xi}{\partial \tau} = \beta\gamma(T-\tau)^{-\beta-1} = \beta\xi(T-\tau)^{-1} \qquad (20)$$

We put (20) to (19) and have the following:

$\frac{\partial W}{\partial \tau} = (T-\tau)^{\alpha-1}\left(-\alpha\, f(\xi) + \beta\xi\frac{df}{d\xi}\right)$ (21)

$W^{\sigma} = (T-\tau)^{\alpha\sigma} f^{\sigma}(\xi)$ (22)

$W_{\gamma} = \frac{\partial}{\partial \gamma}\left((T-\tau)^{\alpha} f(\xi)\right) = \frac{\partial}{\partial \xi}\left((T-\tau)^{\alpha} f(\xi)\right)\frac{\partial \xi}{\partial \gamma} = (T-\tau)^{\alpha}\frac{df}{\partial \xi}\frac{\partial \xi}{\partial \gamma}$ (23)

$\frac{\partial \xi}{\partial \gamma} = (T-\tau)^{-\beta}$ (24)

We shall put (24) to (23). In doing so, the following equality is formed:

$W_{\gamma} = (T-\tau)^{\alpha-\beta}\frac{df}{d\xi}$ (25)

$\frac{\partial}{\partial \gamma}\left(W^{\sigma}W_{\gamma}\right) = \frac{\partial}{\partial \xi}\left(W^{\sigma}W_{\gamma}\right)\frac{\partial \xi}{\partial \gamma} = \frac{\partial}{\partial \xi}\left((T-\tau)^{\alpha\sigma} f^{\sigma}(\xi)(T-\tau)^{\alpha-\beta}\frac{\partial f}{\partial \xi}\right)(T-\tau)^{-\beta} = (T-\tau)^{\alpha(\sigma+1)-2\beta}\frac{\partial}{\partial \xi}\left(f^{\sigma}\frac{\partial f}{\partial \xi}\right)$ (26)

When we put (21), (26) into equation (16), the following approximate self-similar equation is formed:

$$(T-\tau)^{\alpha(\sigma+1)-2\beta}\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right) + (T-\tau)^{\alpha-1}\left(\alpha\, f - \beta\xi\frac{df}{d\xi}\right) = 0 \qquad (27)$$

if we take $\alpha(\sigma+1) - 2\beta = \alpha - 1$, we get $\alpha = (2\beta-1)/\sigma$ for α.

When we put the found α (27) into an approximate self-similar equation, the equation goes to the following view:

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right) + \alpha\, f - \beta\xi\frac{df}{d\xi} = 0 \qquad (28)$$

(28) if we add βf to the equation and subtract

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right) + \alpha\, f + \beta f - \beta\xi\frac{df}{d\xi} - \beta f = 0$$

$$-\beta\xi\frac{df}{d\xi} - \beta f = -\frac{d}{d\xi}(\beta f\xi)$$

represents a full differential. Using such (28), we write down the equation as follows:

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{\partial f}{\partial \xi}\right) - \frac{d}{d\xi}(\beta f\xi) + (\alpha + \beta)f = 0 \qquad (29)$$

In equation (29) $Af = (\alpha+\beta)\, f$ functionality is included.

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right) - \frac{d}{d\xi}(\beta f\xi) + Af = 0 \quad (29)$$

(29) to solve the approximate self-similar equation, we solve this equation without the Af functionality:

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right)-\frac{d}{d\xi}(\beta f\xi)=0 \qquad (30)$$

Equation (30) represents a simple differential equation. This is the solution of the differential equation f we find as follows:

$$\frac{d}{d\xi}\left(f^{\sigma}\frac{df}{d\xi}\right)=\frac{d}{d\xi}(\beta f\xi)$$

$$f^{\sigma}\frac{df}{d\xi}=\beta f\xi$$

$$f^{\sigma-1}df=\beta\xi d\xi$$

$$\frac{f^{\sigma}}{\sigma}=\frac{\beta}{2}\xi^{2}+c_1$$

$$f=\left(c_2+\frac{\sigma\beta}{2}\xi^{2}\right)^{\frac{1}{\sigma}} \qquad (31)$$

Theorema1. If

$$\alpha+\beta\geq 0,$$

$$\beta\geq-\alpha=\frac{-2\beta+1}{\sigma},$$

$$\beta\sigma\geq 1-2\beta, \beta(\sigma+2)\geq 1,$$

$\beta\geq\frac{1}{\sigma+2}$ (32)

if appropriate, the (31) function (29) would be the higher solution for the equation.

Theorema2. If

$$\alpha+\beta\leq 0,$$

$$\beta\leq-\alpha=\frac{-2\beta+1}{\sigma},$$

$$\beta\sigma\leq 1-2\beta, \beta(\sigma+2)\leq 1,$$

$\beta\leq\frac{1}{\sigma+2}$ (33)

if appropriate, the (31) function (29) would be the lower solution for the equation.

Let's look at some private cases for the equation (1) given above:

1) $\rho(t)=t^{\alpha_1}$, $\vartheta(t)=t^{\alpha_1}$, $\tau(0)=0$,

$\tau(t)$ we find the function, in which we find the appearance of $\tau(t)$ in the form of (6).

$$\tau(t)=C+\int\frac{1}{t^{\alpha_1}}dt=C+\frac{t^{1-\alpha_1}}{1-\alpha_1}$$

$\tau(0)=0$ using the condition, we find $C=const$:

$$\tau(0)=C=0$$

It would seem $\tau(t)=\frac{t^{1-\alpha_1}}{1-\alpha_1}$ (34) will like.

$$\vartheta(t)=t^{\alpha_1} \Rightarrow Z(\tau)=\left((1-\alpha_1)\tau\right)^{\frac{\alpha_1}{1-\alpha_1}}$$

$\gamma(\tau,x)$ is defined in the form of a function (11) and this function is as follows:

$\gamma(\tau,x) = x - \int \left((1-\alpha_1)\tau\right)^{\frac{\alpha_1}{1-\alpha_1}} d\tau = x - \left((1-\alpha_1)\tau\right)^{\frac{1}{1-\alpha_1}} = x - t$ (35)

ξ the function was in the form of (18) and according to (34) and (35) (18)

$\xi = (x-t)\left(\mathrm{T} - \frac{t^{1-\alpha_1}}{1-\alpha_1}\right)^{-\beta}$ (36)

it switches to visibility, and $u(t,x)$ while the approximate automodel solution appears to be the following:

$u(t,x) = \left(T - \frac{t^{1-\alpha_1}}{1-\alpha_1}\right)^{\alpha}\left(c_2 + \frac{\sigma\beta}{2}\xi^2\right)^{\frac{1}{\sigma}}$ (37)

2) $\rho(t) = e^{\alpha_1 t}$, $\vartheta(t) = e^{\alpha_1 t}$, $\tau(0) = 0$,

$\tau(t)$ we find the function, in which we find the appearance of τ (t) in the form of (6).

$$\tau(t) = C + \int \frac{1}{e^{\alpha_1 t}} dt = C - \frac{e^{-\alpha_1 t}}{\alpha_1}$$

$\tau(0) = 0$ using the condition, we find $C = const$:

$$\tau(0) = C - \frac{1}{\alpha_1} = 0, \qquad C = \frac{1}{\alpha_1}$$

It would seem $\tau(t) = \frac{1-e^{-\alpha_1 t}}{\alpha_1}$ (38) will like.

$$\vartheta(t) = e^{\alpha_1 t} => Z(\tau) = \frac{1}{1-\alpha_1\tau}$$

$\gamma(\tau,x)$ is defined in the form of a function (11) and this function is as follows:

$\gamma(\tau,x) = x - \int \frac{1}{1-\alpha_1\tau} d\tau = x + \frac{1}{\alpha_1}\ln(1-\alpha_1\tau) = x - t$ (39)

ξ the function was in the form of (18) and according to (38) and (39) (18)

$\xi = (x-t)\left(\mathrm{T} - \frac{1-e^{-\alpha_1 t}}{\alpha_1}\right)^{-\beta}$ (40)

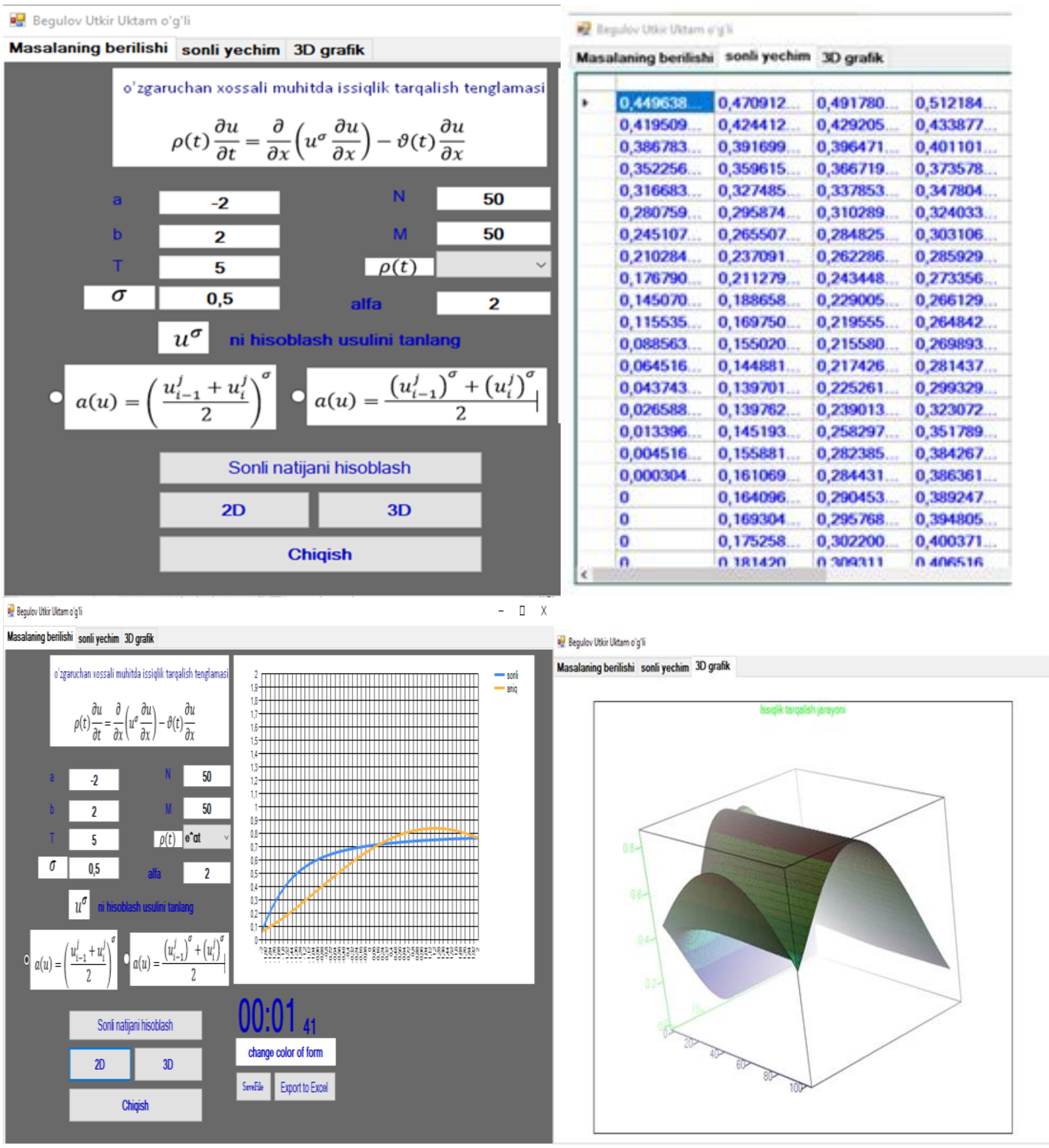
it switches to visibility, and $u(t,x)$ while the approximate automodel solution appears to be the following:

$u(t,x) = \left(T - \frac{1-e^{-\alpha_1 t}}{\alpha_1}\right)^{\alpha}\left(c_2 + \frac{\sigma\beta}{2}\xi^2\right)^{\frac{1}{\sigma}}$ (41)

Calculation exponent and numerical results:

An algorithm for solving the problem posed was compiled and an algorithm-compatible program was created in the C# programming language (Figure 1). Numerical results were obtained (Figure 2), based on the results obtained, two-dimensional (figure 3) and three-dimensional (figure 4) graphic animations were formed. To work with graphics mode, the Chart module of the C# programming language, the 3D-Plot graphics module of the Mathcad package, was used. A user-friendly interface has been created to use the program.

Some of the results generated using the program are reflected below:



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