BAUM-KAS TEOREMALARI VA ULARNI UMUMLASHTIRISH

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Аннотация. Maqolada эҳтимоллардан тузилган қаторларнинг яқинлашиши учун зарурий шартлар келтирилган.

Tayanch so'zlar: ehtimollik, markaziy chegara teoremalari, ko'rsatkichli, ketma-ketliklar yig'indilarining yaqinlashuvi, mustaqillik, normal taqsimot, ekstremal taqsimot, konvergensiya.

Аннотация. В данной статья получена необходимые условия сходимости рядов из вероятностей для сумм случайных величин с многомерными индексами.

Ключевые понятия: вероятность, центральные предельные теоремы, экспоненционал, сходимость сумм последовательностей, независимость, нормальное распределение, экстремальное распределение, сходимость.

Annotation. In this article, necessary conditions for the convergence of series of probabilities for sums of random variables with multidimensional indices are obtained.

Key words: probability, central limit theorem, exponential, convergence of sums of sequences, independence, normal distribution, extremal distribution, convergence .

Z^d_+ - n – o'lchovli arifmetik fizo bo'lib, uning elementlari musbat butun sonlardan iborat bo'lsin. Z^d_+ - da qisman tartiblanganlik tushunchasini kiritamiz: "<" Agar $\overline{m} = (m\ , m\ ... m_d)$ *ва* $\overline{n} = (n\ , n\ .... n_d)$ *бўлиб*, $m_i \le n_i\ , i = \overline{i, d}$ *бўлса* $\overline{m} < \overline{n}$

каби ёзилади. Шунингдек, хар бир i да (i=i, d) $n_i \to \infty$ бўлса, $\overline{n} \to \infty$ бўлади.

Teorema 1. X *ва* $\{X(\overline{n}), \overline{n} \in Z^d_+\}$ bog'liq bo'lmagan va normal taqsimlangan tasodifiy miqdorlar bo'lib, ular uchun quyidagi shart bajarilsin:

$EX(\overline{n}) = 0,\ EX^2(\overline{n}) = 1$ *ва* $S(\overline{n}) = \sum_{k<n} X(\overline{n})$, shuningdek

$\sigma^2(n) = VarX \cdot I\{\ X\ | \le \sqrt{|\ n\ |}\}$ bo'lsin, bu yerda I{.} –hodisaning indikatori.

Agar $EX^{2(2-\alpha)} \exp(X)(\log^+ |\ X\ |)^{d+\beta-1} < \infty$ bo'lsa, u holda

$$\sum_{\bar{n}} \exp(|n|)\frac{(\ln n)^{\beta}}{|\bar{n}|^{\alpha}} Sup\left|P(\frac{S(\bar{n})'}{\sqrt{|n|}} \le X) - \phi(\frac{x}{\sigma_n})\right| < \infty \quad (1)$$

bo'ladi, bu yerda $\log^{+}x=\max(0, \log x)$

Teoremaning isboti.

Teoremani isbotlash uchun sonlar nazariyasining quyidagi natijalaridan foydalanamiz.

$$d(j) = card\{\bar{k}, |\bar{k}| = j\}$$
$$\hat{a}\grave{a} M(j) = card\{\bar{k}, |\bar{k}| \le j\}$$

[14] ga asosan $j \to \infty$

$$M(j) \approx \frac{j(\log j)^{d-1}}{(d-1)!} \text{ bo'ladi.} \quad (2)$$

Shuningdek, $\forall \delta > 0 \ \ ва \ \ j \to \infty \ да$ $d(j) = 0(j^{b})\ бўлади$

Gut [3] ga asosan .

Teoremani isbotlashda quyidagi lemmalardan [2] foydalanamiz.

Lemma.1. $k \to \infty$ da quyidagi munosabatlar o'rinli bo'ladi:

$$\sum_{j=1}^{k} d(j) j^{\alpha} \le C k^{\alpha+1} (\log x)^{d-1}; \ (j > -1) \ (4)$$

$$\sum_{j=1}^{k} \frac{d(j)(\log j)^{\delta}}{j} \le C \cdot (\log k)^{d+\delta}; \ (\delta \ge -1) \ (5)$$

$$\sum_{j=1}^{k} \frac{d(j)(\log j)^{\delta}}{j^{\alpha}} \le C \cdot \frac{(\log k)^{d-1+\delta}}{(\gamma-1)k^{\gamma-1}}. \ (6)$$

$$(\gamma > 1, -\infty < \delta < \infty)$$

Lemma 2. ξ - nomanfiy tasodifiy miqdor bo'lsa, u holda r>0 uchun quyidagi munosabat o'rinli:

$$\sum_{j=1}^{\infty} d(j)\, j^{r-1} P(\xi > j) < \infty \Leftrightarrow E\xi^{r} (\log^{+}\xi)^{d-1} < \infty$$

Endi (1) munosabatni isbot qilamiz:

(1) munosabatni isbotlash uchun quyidagi qatorni yaqinlashtiruvchiligini isbotlash yetarli

$$\sum{}_0 = \sum_{j=1}^{\infty} \frac{\exp(j)(\ln j)^{\beta} d(j)}{j^{\alpha}} \underset{\tilde{o}}{S\,up} \left| \; P\left(\frac{S(\pi(j))}{\sqrt{j}} \le x\right) - \phi\left(\frac{x}{\sigma_j}\right) \right| < \infty$$ Куйидаги

tengsizlikdan foydalanamiz [1] :

$$\left| P\left(\frac{S\pi(j)}{\sqrt{j}} \le x\right) - P\left(\frac{S_j^{'}}{\sqrt{j}} \le x\right) \right| \le jP(|x| > \sqrt{j}) \qquad (7)$$

tengsizlik o'rinli. Bu tengsizlikka asosan:

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha}} \sup_x \left| P\left(\frac{S\pi(j)}{\sqrt{j}} \le x\right) - \phi\left(\frac{x}{\sigma_j}\right) \right| \le$$

$$\le \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha-1}} \left| P(|X| > \sqrt{j}) + \right.$$

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha}} \sup_x \left| P\left(\frac{S_j^{'}}{\sqrt{j}} \le x\right) - \phi\left(\frac{x - \mu_i \sqrt{j}}{\sigma_j}\right) \right| +$$

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha}} \sup_x \left| \phi\left(\frac{x - \mu_i \sqrt{j}}{\sigma_j}\right) - \phi\left(\frac{x}{\sigma_j}\right) \right| =$$

$$\sum{}_1 + \sum{}_2 + \sum{}_3 .$$

$\sum_1, \sum_2, \sum_3$ larni baholaymiz.

$$\sum_1 = \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha-1}} P(|X| > \sqrt{j}) =$$

$$= \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha-1}} \sum_{k=j}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) =$$

$$= \sum_{k=1}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) \sum_{j=1}^{k} \frac{(\ln j)^{\beta} \exp() j d(j)}{j^{\alpha-1}} =$$

$$= \sum_{k=1}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) \sum_{j=1}^{k} j^{\alpha-1} (\ln j)^{\beta} \exp(j) d(j) \le$$

$$\le C \cdot \sum_{k=1}^{\infty} \hat{e}^{2-\alpha} \exp(k) (\log^{+} k)^{\beta+d-1} P(\sqrt{k} < |X| < \sqrt{k+1}) \le$$

$$\le C \cdot E|X|^{2(2-\alpha)} \exp(X) (\log^{+}|X|)^{\beta+d-1} < \infty$$

$\sum_2$ ni baholaymiz.

Essen tengsizlikka ko'ra [2]

$$\sup_x \left| P\left(\frac{s_j'}{\sqrt{j}} \le x\right) - \phi\left(\frac{x - \mu_j \sqrt{j}}{\sigma_j}\right) \right| \le$$

$$\le C \frac{E|Y_{ij} - \mu_j|^3}{\sqrt{j} \sigma_j^3} \le C \frac{8E|Y_{ij}|^3}{\sqrt{j} \sigma_j^3}$$

Bu yerdan (6) ga asosan,

$$\sum_2 \le C \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha}} \cdot \frac{E|Ó_{ij}|^3}{\sqrt{j} \sigma_j^3} \le C + C \sum_{j \ge j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha + \frac{1}{2}}} \le$$

$$\le C + C \sum_{j \ge j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha + \frac{1}{2}}} \int_{|x| = \sqrt{j}} |x|^3 \, dF(x) \le$$

$$\le C + C \int_0^{\infty} \left(\sum_{j \ge x^2} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha} + \frac{1}{2}} \right) |x|^3 \, dx \le$$

$$\int_0^{\infty} x^{2(2-\alpha)} exp(x) (\log^{+}|x|)^{\beta+d-1} dF(x) < \infty,$$

Teorema shartiga asosan yaqinlashuvchi.

Endi $\sum_3$ ni baholaymiz.

1-lemmaga asosan va teorema shartlariga ko'ra

$$\sum_3 \leq \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha}} \cdot \frac{1}{\sqrt{2\pi}} \cdot \frac{|\mu_i| \sqrt{j}}{\sigma_j} \leq$$

$$\leq \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha - \frac{1}{2}}} \cdot \frac{1}{\gamma_j} \int_{|x| \geq \sqrt{j}} |x| \, dF(x) \leq$$

$$\leq 2 + 2 \frac{1}{\sqrt{2\pi}} \sum_{j \geq j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j) d(j)}{j^{\alpha - \frac{1}{2}}} \int_{|x| \geq \sqrt{j}} |x| \, dF(x) \leq$$

$$C + C \int_0^{\infty} (x^{2(2-\alpha)} \exp(x) (\log^{+} |x|)^{\beta + d - 1} dF(x) < \infty$$

Bo'ladi.Teorema to'la isbotlandi.

Izoh. Isbotlangan teoremadan xususiy holda, ya'ni $\alpha = 1$ bo'lganda [2] dagi teorema kelib chiqadi.

Адабиётлар

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