KUCHAYTIRILGAN KATTA SONLAR QONUNI VA UNDAGI YAQINLASHISH TEZLIKLARI

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Annotasiya. Ushbu maqolaning maqsadi U.Stadtmyuller ishlarni ko'rib chiqishdir va klassik katta sonlar qonunlarining tasodifiy maydon analoglari haqida. Makolada tasodifiy maydonlar uchun va ketma-ketliklar uchun karrali logarifmlar qonuni uchun zaruriy shartlar keltirilgan.

Аннотация. Целью данной статьи является обзор работы У. Штадтмюллера и аналогов случайных полей классических законов больших чисел. В статье представлены необходимые условия закона кратных логарифмов для случайных полей и для последовательностей.

Ключевые понятия: вероятность, центральные предельные теоремы, экспоненционал, среднеквадратическое отклонельныение, коэффициент вариации, доверительная интервал,

Abstract. The purpose of this article is to review the work of W. Stadtmüller and analogues of random fields of the classical laws of large numbers. The article presents the necessary conditions for the law of multiple logarithms for random fields and for sequences.

Key words: Multidimensional indices; Tail probabilities of sums of i.i.d. random variables; Stable distributions; Domain of attraction; Strong law; Law of the iterated logarithm.

Kirish. Ehtimollar nazariyasi va matematik statistika fanining XX asrdagi rivoji va uning xalq xo'jaligining turli tarmoqlariga tadbiqlari o'zining ilmiy amaliy ahamiyati bilan ajralib turadi. Buning misoli sifatida A.N.Kolmogorov, A.Xinchin, Yu.V.Proxorov, B.V.Gnedenko, A.Borovkov, A.N.Shiryayev, V.Petrov, V.Korolyuk, S.X.Sirojiddinov, A.Skoroxod, Y.P.Kubilyus, V.Feller kabi olimlarning ishlari fundamentalligi bilan ajralib turadi.

Ehtimollardan tuzilgan qatorlarning yaqinlashishini o'rganish keyingi yillardagi muhim yo'nalishlardan xisoblanadi. Ana shunday natijalarga A.Gut, O.Klesov, Husler va Klesov, A.Bulinskiy, A. Spataru, U.Shtadtmyuller kabilarning ishlarida indekslari ko'p o'lchovli tasodifiy miqdorlar uchun qator fundamental natijalar olindi. Olingan natijalar shuni ko'rsatadiki, ehtimollardan tuzilgan qatorlarning yaqinlashish katta sonlar qonuni, chegaraviy funksionallar taqsimoti (chegaraviy shartlar bo'yicha chegaradan chiqishlar soni, ekssess va oxirgi chiqish momenti) kabi masalalarni yechishda keng ko'lamda qo'llanilgan. Maqola asosan nazariy xarakterga ega bo'lib, uning natijalaridan iqtisodiyotda va ilmiy texnika fanlarida qo'llanilishi mumkin.

Biz R orqali xaqiqiy sonlar to`plamini, N bilan natural sonlar to'plamini belgilaymiz. Faraz qilaylik, quyidagi d-o'lchovli koordinatalari natural sonlardan iborat vektorlar to`plami berilgan bo`lsin:

$$Z_+^d = \{\vec{n} = (n_1, n_2, \dots, n_d);\ n_i \epsilon N, i = \overline{1, d},$$

bu yerda $d\epsilon N$. Z_+^d to`plamda qisman tartiblanganlik tushunchasini kiritamiz: $\vec{m}, \vec{n} \in Z_+^d$ uchun $\vec{m} < \vec{n}$ munosabat $m_i < n_i, i = \overline{1, d},$ ni bildiradi.

Ushbu $\vec{n} = (n_1, n_2, \dots, n_d) \in Z_+^d$ uchun

$$|\bar{n}| = \prod_{i=1}^{d} n_i$$

bo`lsin, agar $n_i \to \infty$ har bir, $i = \overline{1, d}$ uchun bo`lsa, $\bar{n} \to \infty$ ni bildiradi. Quyidagi munosabat o`rinli $\overline{\alpha n} = (\alpha n_1, \dots, \alpha n_d), \alpha \in R,$ $log^+ x = \max(0, logx)$ belgilashni kiritamiz.

Kelgusida $\{X(\vec{n});\ n \in Z_+^d$ orqali bog`liq bo`lmagan, bir hil taqsimlangan tasodifiy miqdorlar to`plamini belgilaymiz:

$$S(\bar{0}) = 0, \quad S(\bar{n}) = \sum_{k<n} X(\bar{k})$$

X orqali $X(\bar{n});\ n \in Z_+^d$ tasodifiy miqdor bilan bir hil taqsimotga ega bo`lgan tasodifiy miqdorni belgilaymiz.

Ushbu yo`nalishdagi birinchi natijalar Husler[1] va Klesov[1, 2] tomonidan keltirilgan. Ular yuqoridagilarni natijasini umumlashtirib: $EX = 0$ va $E[X^2(\log(1 + |X|))^{d-1}] < \infty$ (1) bo`lsa, u holda

$$\lim_{\varepsilon\to 0} \frac{\varepsilon^2}{(-2log\varepsilon)^{d-1}} \sum_n P(|S_n| \geq \varepsilon|n|) = \frac{EX^2}{(d-1)!}, \qquad (2)$$

bo'lishini ko'rsatdi.

Ushbu maqolada quyidagi teorema isbot qilingan.

Teorema. Faraz qilaylik $EX = 0$, u holda $E[|X|^r(\log(1 + |X|))^{d-1}] < \infty$, $r \geq 2$, $EX^2 = 1$ ni olamiz va N standart normal taqsimlangan tasodifiy miqdor bo`lsin. $1 \leq p < 2$ uchun

$$\lim_{\varepsilon\to 0}\frac{\varepsilon^{\frac{2p}{2-p}\left(\frac{r}{p}-1\right)}}{(-log\varepsilon)^{d-1}}\sum_{n}|n|^{\frac{r}{p}-2}P\left(|S_n|\geq\varepsilon|n|^{\frac{1}{p}}\right)=$$
$$=\frac{1}{(d-1)!}\left(\frac{2p}{2-p}\right)^{d-1}\cdot\frac{p}{r-p}E|N|^{\frac{\propto p}{\propto -p}\left(\frac{r}{p}-1\right)},$$

Natija. Teoremadan r=2 va ∝→ 2 bo`lganda yuqoridagi yaqinlashayotganini ko`rishimiz mumkin.

Teoremaning isboti.

Teoremani isbotlashdan avval quyidagi lemmalarni ko`rib o`tamiz.

Shunday qilib, biz normallanganligi uchun uning dispersiyasi $\sigma^2 = 1$ni bilamiz. Bundan tashqari, N standart oddiy tasodifiy miqdor, bu kichik bo'limda F Φ ning taqsimot funksiyasi va $\Psi(x) = 1 - \Phi(x) + \Phi(-x) = (|N| > x),\ x \geq 0$.

Lemma-1. $d \geq 2\ va\ \gamma > 0$ lar uchun

$$\lim_{\varepsilon\to 0}\frac{1}{(-log\varepsilon)^{d-1}}\int_{\varepsilon C}^{\infty}\left(\log\frac{y}{\varepsilon}\right)^{d-1}y^{\gamma-1}\Psi(y)dy=\gamma^{-1}E|Z|^{\gamma}.$$

Isbot. Lemma 2.4 dan darhol $h(y) = y^{\gamma-1}\,\Psi(y)$ bilan isboti kelib chiqadi.

Lemma -2. $k \to \infty$ da $\gamma > -1$ uchun biz

$$\sum_{j=1}^{k}d(j)j^{\gamma}\sim\frac{1}{(d-1)!}\sum_{j=1}^{k}j^{\gamma}\,(logj)^{d-1}\sim\frac{1}{(d-1)!}\frac{k^{\gamma+1}(logk)^{d-1}}{\gamma+1}.$$

-ga ega bo'lamiz.

Isbot. $k > 1$ uchun biz $\sum_{j=1}^{k}d(j)j^{\gamma} = M(k)k^{\gamma} + \sum_{j=1}^{k-1}d(j)(j^{\gamma} - k^{\gamma})$ tenglikni olamiz. Keyin $\sum_{j=1}^{k}d(j)j^{\gamma}$ yig`indini $M(k)k^{\gamma} - \gamma\sum_{i=1}^{k-1}M(i)i^{\gamma-1}$ va $M(k)k^{\gamma} - \gamma\sum_{i=1}^{k-1}M(i)(i+1)^{\gamma-1}$ lar orasida yotishini kuzatamiz. Bu esa $j^{\gamma} - k^{\gamma}$ ning $-\gamma\sum_{i=j}^{k-1}i^{\gamma-1}$ va $-\gamma\sum_{i=j}^{k-1}(i+1)^{\gamma-1}$ lar orasida yotishini anglatadi. $\sum_{j=1}^{k}j^{\gamma}(\log j)^{d-1}\sim\frac{k^{\gamma+1}(\log k)^{d-1}}{\gamma+1}$ ning $k \to \infty$ ekanligini hisobga olib, biz quyidagini hosil qilamiz:

$$M(k)k^{\gamma}-\gamma\sum_{i=1}^{k-1}M(i)i^{\gamma-1}\sim\frac{k^{\gamma+1}(\log k)^{d-1}}{(d-1)!}-$$

$$-\frac{\gamma}{(d-1)!}\sum_{i=1}^{k-1} i^{\gamma}(\log i)^{d-1} \sim \frac{\gamma+1}{(d-1)!}\sum_{j=1}^{k} j^{\gamma}(logj)^{d-1} - \frac{\gamma}{(d-1)!}\sum_{i=1}^{k-1} i^{\gamma}(logi)^{d-1}$$

$$= \frac{1}{(d-1)!}\sum_{j=1}^{k} j^{\gamma}(logj)^{d-1} + \frac{\gamma}{(d-1)!}k^{\gamma}(logk)^{d-1}$$

$$\sim \frac{1}{(d-1)!}\sum_{j=1}^{k} j^{\gamma}(logj)^{d-1} \sim \frac{1}{(d-1)!}\frac{k^{\gamma+1}(\log k)^{d-1}}{\gamma+1}.$$

Bog`langan $M(k)k^{\gamma} - \gamma\sum_{i=1}^{k-1} M(i)(i+1)^{\gamma-1}$ $k \to \infty$ intilganda bir xil asimptotikani ta'minlaydi.

Tasdiq. $r \geq 2$ va $1 \leq p < 2$ uchun

$$\lim_{\varepsilon\to 0}\frac{\varepsilon^{\frac{\alpha p}{\alpha-p}\left(\frac{r}{p}-1\right)}}{(-log\varepsilon)^{d-1}}\sum_{j\geq 1} d(j) j^{\frac{r}{p}-2} P\left(\left|S_{\pi(j)}\right| \geq \varepsilon j^{\frac{1}{p}}\right) =$$

$$= \frac{1}{(d-1)!}\left(\frac{2p}{2-p}\right)^{d-1}\frac{p}{r-p}E|N|^{\frac{2p}{2-p}\left(\frac{r}{p}-1\right)}.$$

Isbot. $\frac{r}{p} - 2$ uchun xulosa 4.1-tasdiqdagi kabi α o`rniga 2 gani, Z esa N bilan almashtiriladi. Shuning uchun $r \geq 2p$ bo'lsin. $\gamma = \frac{r}{p} - 2$ bilan Lemma -2. bo'yicha, k_0 ni $k \geq k_0$ shunday tanlaymiz ya'ni $0 < \delta < 1$ uchun;

$$\frac{1-\delta}{(d-1)!}\sum_{j=1}^{k}(logj)^{d-1} j^{\frac{r}{p}-2} \leq \sum_{j=1}^{k} d(j)\, j^{\frac{r}{p}-2} \leq \frac{1+\delta}{(d-1)!}\sum_{j=1}^{k}(logj)^{d-1}\, j^{\frac{r}{p}-2},$$

Bundan tashqari, $j \geq k_0$ uchun $(logj)^{d-1} j^{\frac{r}{p}-2} \leq (1+\delta)(\log(j-1))^{d-1}(j-1)^{\frac{r}{p}-2}$ deb faraz qilamiz. Lemmalarga asosan quyidagini olamiz.

$$\sum_{j\geq 1} d(j) j^{\frac{r}{p}-2} P\left(\left|S_{\pi(j)}\right| \geq \varepsilon j^{\frac{1}{p}}\right) \leq C + \frac{1+\delta}{(d-1)!}\sum_{j\geq k_0}(logj)^{d-1} j^{\frac{r}{p}-2}\Psi\left(\varepsilon j^{\frac{1}{p}-\frac{1}{2}}\right)$$

$$\leq C + \frac{(1+\delta)^2}{(d-1)!}\sum_{j\geq k_0}(\log(j-1))^{d-1}(j-1)^{\frac{r}{p}-2}\Psi\left(\varepsilon j^{\frac{1}{p}-\frac{1}{2}}\right)$$

$$\leq C + \frac{(1+\delta)^2}{(d-1)!}\int_{k_0-1}^{\infty}(\log x)^{d-1} x^{\frac{r}{p}-2}\Psi\left(\varepsilon x^{\frac{1}{p}-\frac{1}{2}}\right) dx$$

$$\leq C + \varepsilon^{-\frac{2p}{2-p}\left(\frac{r}{p}-1\right)} \frac{(1+\delta)^2}{(d-1)!}\left(\frac{2p}{2-p}\right)^d \int_{\varepsilon C}^{\infty}\left(\log\frac{y}{\varepsilon}\right)^{d-1} y^{\frac{2p}{2-p}\left(\frac{r}{p}-1\right)}\Psi(y)\,dy\,,$$

bu va $\gamma = \frac{2p}{2-p}\left(\frac{r}{p}-1\right)$ bilan Lemma 2 dan quyidagi natia kelb chiqadi:

$$\lim_{\varepsilon\to 0}\sup \frac{\varepsilon^{\frac{2p}{2-p}\left(\frac{r}{p}-1\right)}}{(-\log\varepsilon)^{d-1}}\sum_{j\geq 1} d(j)\, j^{\frac{r}{p}-2} P\left(\left|S_{\pi(j)}\right| \geq \varepsilon j^{\frac{1}{p}}\right) =$$

$$\frac{1}{\sqrt{\pi}}\cdot\frac{1}{(d-1)!}\cdot\left(\frac{2p}{2-p}\right)^{d-1}\cdot\frac{p}{r-p}\cdot 2^{\frac{\alpha p}{\alpha-p}\cdot\left(\frac{2}{p}-1\right)}\Gamma\left(\frac{\alpha p}{\alpha-p}\cdot\left(\frac{2}{p}-1\right)+\frac{1}{2}\right).$$

$$\frac{1}{\sqrt{\pi}}\cdot\frac{1}{(d-1)!}\cdot\left(\frac{2p}{2-p}\right)^{d-1}\cdot\frac{p}{r-p}\cdot 2^{\frac{\alpha p}{\alpha-p}\cdot\left(\frac{2}{p}-1\right)}\Gamma\left(\frac{\alpha p}{\alpha-p}\cdot\left(\frac{2}{p}-1\right)+\frac{1}{2}\right)$$

yuqoridagilardan teoremani isboti kelib chiqadi.

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