INDEKSLARI KO'P O'LCHOVLI TASODIFIY MIQDORLAR YIG'INDISI UCHUN LIMIT TEOREMALAR

F.M.Xolmurodov
[1]Namangan davlat universiteti dotsenti

A.A.Asqarov
Namangan davlat universiteti magistranti
asqarovabdullo007@gmail.com

Annotatsiya. Maqolada ehtimollardan tuzilgan qatorlarning yaqinlashishi uchun zaruriy shartlar keltirilgan.

Tayanch so'zlar: ehtimollik, markaziy chegara teoremasi, eksponensial, ketma-ketliklar yig'indilarining yaqinlashuvi, mustaqillik, normal taqsimot, ekstremal taqsimot, yaqinlashish .

Аннотация. В данной статья получена необходимые условия сходимости рядов из вероятностей для сумм случайных величин с многомерными индексами.

Ключевые понятия: вероятность, центральные предельные теоремы, экспоненционал, сходимость сумм последовательностей, независимость, нормальное распределение, экстремальное распределение, сходимость.

Abstract. The article presents the necessary conditions for the convergence of probabilistic series.

Key words: probability, central limit theorem, exponential, convergence of sums of sequences, independence, normal distribution, extremal distribution, convergence .

Z^d_+ - n –o'lchovli arifmetik fazo bo'lib, uning elementleri musbat butun sonlardan iborat bo'lsin. Z^d_+ - da qisman tartiblanganlik tushunchasini kiritamiz: "<" Agar

$$\overline{m} = (m\ ,m\ ...m_d)\ \ va\ \overline{n} = (n\ ,n\ ....n_d)\ bo'lib,\ m_i \le n_i\ ,i = \overline{i,d}\ \ bo'lsa\ \ \overline{m} < \overline{n}$$

ka'bi yoziladi. Shuningdek har bir i da (i=i, d) $n_i \to \infty$ bo'lsa, $\overline{n} \to \infty$ bo'ladi.

Teorema 1. $X\ va\ \left\{X(\overline{n}), \overline{n} \in Z^d_+\right\}$ bog'liq bo'lmagan va normal taqsimlangan tasodifiy miqdorlar bo'lib, ular uchun quyidagi shart bajarilsin:

$$EX(\overline{n}) = 0,\ \ EX^2(\overline{n}) = 1\ \ ва\ \ S(\overline{n}) = \sum_{k<n} X(\overline{n}),$$

shuningdek $\sigma^2(n) = VarX \cdot I\left\{\,|X| \le \sqrt{|n|}\right\}$ bo'lsin, bu yerda I{.} – hodisaning indikatori.

Agar $EX^{2(2-\alpha)}\exp(X^{\lambda})(\log^{+}|X|)^{d+\beta-1}<\infty$ bo'lsa, u holda

$$\sum_{\bar{n}}\frac{\exp(|n|^{\lambda})(\ln)^{\beta}}{|\bar{n}|^{\alpha}}Sup\left|P(\frac{S(\bar{n})'}{\sqrt{|n|}}\leq X)-\varphi(\frac{x}{\sigma_n})\right|<\infty \qquad (1)$$

bo`ladi, bu yerda $\log^{+}x=\max(0,\log x)$.

Teoremaning isboti.

Teoremani isbotlash uchun sonlar nazariyasining quyidagi natijalaridan foydalanamiz.

$$d(j)=card\{\bar{k},|\bar{k}|=j\}$$
$$\hat{a}\grave{a}M(j)=card\{\bar{k},|\bar{k}|\leq j\}$$

[4] ga asosan $j\to\infty$

$$M(j)\approx\frac{j(\log j)^{d-1}}{(d-1)!}$$ bo'ladi. (2)

Shuningdek, $\forall\delta>0$ *ва* $j\to\infty$ *da* $d(j)=0(j^{b})\,bo'ladi$

Gut [3] ga asosan.

Teoremani isbotlashda quyidagi lemmalardan[2] foydalanamiz.

Lemma.1. $k\to\infty$ da quyidagi munosabatlar o'rinli bo'ladi:

$$\sum_{j=1}^{k}d(j)j^{\alpha}\leq Ck^{\alpha+1}(\log x)^{d-1};\ (j>-1)\ (4)$$

$$\sum_{j=1}^{k}\frac{d(j)(\log j)^{\delta}}{j}\leq C\cdot(\log k)^{d+\delta};\ (\delta\geq-1)\ (5)$$

$$\sum_{j=1}^{k}\frac{d(j)(\log j)^{\delta}}{j^{\alpha}}\leq C\cdot\frac{(\log k)^{d-1+\delta}}{(\gamma-1)k^{\gamma-1}}.\ (6)$$

$$(\gamma>1,-\infty<\delta<\infty)$$

Lemma 2. ξ -nomanfiy tasodifiy miqdor bo'lsa, u holda r>0 uchun quyidagi munosabat o'rinli:

$$\sum_{j=1}^{\infty} d(j)\, j^{r-1} P(\xi > j) < \infty \Leftrightarrow E\xi^{r} (\log^{+} \xi)^{d-1} < \infty$$

Endi (1) munosabatni isbot qilamiz:
(1) munosabatni isbotlash uchun quyidagi qatorni yaqinlashtiruvchiligini isbotlash yetarli

$$\sum{}_0 = \sum_{j=1}^{\infty} \frac{\exp(j^{\lambda})(\ln j)^{\beta} d(j)}{j^{\alpha}} \underset{x}{Sup} \left| \; P(\frac{S(\pi(j))}{\sqrt{j}} \le x) - \varphi(\frac{x}{\sigma_j}) \right| < \infty$$

Quyidagi tengsizlikdan foydalanamiz[1] :

$$| P(\frac{S\pi(j)}{\sqrt{j}} \le x) - P(\frac{S_j^{'}}{\sqrt{j}} \le x) | \le jP(| x | > \sqrt{j}) \qquad (7)$$

Tengsizlik o'rinli.
Bu tengsizlikka asosan:

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha}} \sup_{x} | P(\frac{S\pi(j)}{\sqrt{j}} \le x) - \phi(\frac{x}{\sigma_j}) | \le$$

$$\le \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha-1}} | P(| X | > \sqrt{j}) +$$

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha}} \sup_{x} | P(\frac{S_j^{'}}{\sqrt{j}} \le x) - \phi(\frac{x - \mu_i \sqrt{j}}{\sigma_j}) | +$$

$$\sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha}} \sup_{x} | \phi(\frac{x - \mu_i \sqrt{j}}{\sigma_j} - \phi(\frac{x}{\sigma_j}) | =$$

$$\sum{}_1 + \sum{}_2 + \sum{}_3 .$$

$\sum{}_1, \sum{}_2, \sum{}_3$ larni baholaymiz.

$$\sum\nolimits_1 = \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha-1}} P(|X| > \sqrt{j}) =$$

$$= \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha-1}} \sum_{k=j}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) =$$

$$= \sum_{k=1}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) \sum_{j=1}^{k} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) j d(j)}{j^{\alpha-1}} =$$

$$= \sum_{k=1}^{\infty} P(\sqrt{k} < |X| < \sqrt{k+1}) \sum_{j=1}^{k} j^{\alpha-1} (\ln j\ )^{\beta} \exp(j^{\lambda}) d(j) \leq$$

$$\leq C \cdot \sum_{k=1}^{\infty} \kappa^{2-\alpha} \exp(k^{\lambda}) (\log^{+} k)^{\beta+d-1} P(\sqrt{k} < |X| < \sqrt{k+1}) \leq$$

$$\leq C \cdot E|X|^{2(2-\alpha)} \exp(X^{\lambda}) (\log^{+}|X|)^{\beta+d-1} < \infty$$

$\sum_2$ ni baholaymiz.

Essen tengsizlikka ko'ra [2]

$$\sup_x \left| P\left(\frac{s_j'}{\sqrt{j}} \leq x\right) - \phi\left(\frac{x - \mu_j \sqrt{j}}{\sigma_j}\right) \right| \leq$$

$$\leq C \frac{E|Y_{ij} - \mu_j|^3}{\sqrt{j} \sigma_j^3} \leq C \frac{8E|Y_{ij}|^3}{\sqrt{j} \sigma_j^3}$$

Bu yerdan (6) ga asosan,

$$\sum\nolimits_2 \leq C \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha}} \cdot \frac{E|Y_{ij}|^3}{\sqrt{j} \sigma_j^3} \leq C + C \sum_{j \geq j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha + \frac{1}{2}}} \leq$$

$$\leq C + C \sum_{j \geq j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha + \frac{1}{2}}} \int_{|x| = \sqrt{j}} |x|^3 \, dF(x) \leq$$

$$\leq C + C \int_0^{\infty} \left(\sum_{j \geq x^2} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha} + \frac{1}{2}} \right) |x|^3 \, dx \leq$$

$$\int_0^{\infty} x^{2(2-\alpha)} exp(x^{\lambda}) (\log^{+}|x|)^{\beta+d-1} dF(x) < \infty,$$

Teorema shartiga asosan yaqinlashuvchi.

Endi $\sum_3$ ni baholaymiz.

1-lemmaga asosan va teorema shartlariga ko'ra

$$\sum\nolimits_3 \le \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha}} \cdot \frac{1}{\sqrt{2\pi}} \cdot \frac{|\mu_i| \sqrt{j}}{\sigma_j} \le$$

$$\le \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha - \frac{1}{2}}} \cdot \frac{1}{\gamma_j} \int_{|x| \ge \sqrt{j}} |x| \, dF(x) \le$$

$$\le 2 + 2 \frac{1}{\sqrt{2\pi}} \sum_{j \ge j_0}^{\infty} \frac{(\ln j)^{\beta} \exp(j^{\lambda}) d(j)}{j^{\alpha - \frac{1}{2}}} \int_{|x| \ge \sqrt{j}} |x| \, dF(x) \le$$

$$C + C \int_0^{\infty} (x^{2(2-\alpha)} \exp(x^{\lambda}) (\log^{+} |x|)^{\beta + d - 1} dF(x) < \infty$$

Bo'ladi.

Teorema to'la isbotlandi.

Teorema 2. X va $\{X(\bar{k}), \bar{k} \in Z_{+}^{d}\}$ bog'liq bo'lmagan va bir hil taqsimlangan tasodifiy miqorlar bo'lsin. Quyidagi belgilashni kiritamiz.

$S(\bar{n}) = Z_{k<n} X(\bar{n}), \bar{n} \in Z$.

U holda quyidagilar teng kuchli bo'ladi:

A) $$\sum_{\bar{n}} \frac{\exp(n^{\lambda}) \log^{\beta} n}{|\bar{n}|^{a}} \sup \left| P\left(\frac{S(\bar{n})}{\sqrt{|\bar{n}|}}\right) \le x) - \Phi(x) \right| < \infty$$

Б) $$\sum_{\bar{n}} \frac{\exp(n^{\lambda}) \log^{\beta} n}{|\bar{n}|^{a}} (1 - \sigma^{2}_{\bar{n}}) < \infty;$$

В) $$E|X|^{2(2-a)} \exp(X^{\lambda}) (\log^{+} |X|^{\beta + d}) < \infty .$$

Bu teorema $(a = 1.\beta = 0)$ bo'lganda [3] .

Bu teoremaning isboti oldingi teorema ka'bi bajariladi.

Adabiyotlar

1. Сиражиддинов С.Х., Гафуров М.У., «Метод рядов в граничных задачах для случайных блужданий». Ташкент.1987,140 с.
2. Gut A. Convergence rates for probabilities of moderate deviationsfor sums of random variables with multidimensional indices // Ann. Probability. – 1980. – V.8. – P. 298-313.

3. Cut.A.Convergance rates in the central limit theorem for multidimensionally indexed random variables. U.U.D.M.report.2007.
4. M.K. Holmurodov. On probabilistic one-sided deviations for sums of independent random variables with multidimensional indices. Fan. Tashkent. 1985. 207-215.
5. A. Gut, Marcinkiewicz laws and convergence rates in the law of large numbers for random variables withmultidimensional indices, Ann. Probab. 6 (1978) 469–482.
6. A. Gut, Strong laws for independent identically distributed random variables indexed by a sector, Ann. Probab. 11 (1983) 569–577.
7. A. Gut, A. Spa˘taru, Precise asymptotics in the Baum–Katz and Davis law of large numbers, J. Math. Anal. Appl. 248 (2000) 233–246.
8. A. Gut, A. Spa˘taru, Precise asymptotics in the law of the iterated logarithm, Ann. Probab. 28 (2000) 1870–1883.